Assign -5

$f(x,y) = e^{-(x+y)} \quad x>0 \;, y>0$

$P(1<x+y<2)$

~~$P(x+y>1)$~~

$P(0<x+y<2) - P(0<x+y<1)$

$$\Rightarrow \int_0^{\infty}\int_0^{2-x} e^{-(x+y)}\,dx\,dy - \int_0^{\infty}\int_0^{1-x} e^{-(x+y)}\,dx\,dy$$

$$\Rightarrow \int_0^{2}\int_y^{2} e^{-t}\,dt\,dx - \int_0^{\infty}\int_y^{1} e^{-t}\,dt\,dx$$

$$\Rightarrow \int_0 -e^{-t}\Big|_y^{2} - \int_0 -e^{-t}\Big|_y^{1}$$

$$\Rightarrow \int_0 e^{-y} - e^{-2} - \int_0 e^{-y} - e^{-1}$$

$$\Rightarrow \cancel{-e^{-y} - e^{-2}y + e^{-y} + e^{-1}y}$$

$$-e^{-y} - e^{-2}y\Big|_0^{2} + e^{-y} + e^{-1}y\Big|_0^{1}$$

$$\Rightarrow -e^{-2} - 2e^{-2} + 1 + e^{-1} + e^{-1} - 1$$

$$2e^{-1} - 3e^{-2} \quad A$$

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b $E(X/Y=10) = \int_{1/6}^{1} x \frac{f(x,y)}{f_y(y)} \, dx$

$\Rightarrow \int_{1/6}^{1} x \cdot \frac{8xy}{4y(1-y^2)} \, dx = \frac{8y}{4y(1-y^2)} \left. \frac{x^3}{3} \right|_{1/6}^{1}$

$\frac{2}{3(1-y^2)} \left[1 - \frac{1}{216} \right] = \frac{430}{216(1-y^2)}$

$\Rightarrow \frac{430}{216 \times 3 \left(1 - \frac{1}{36}\right)}$

$= \frac{\cancel{430}\,43}{\cancel{216}\,6 \times 3 \times \frac{\cancel{35}\,7}{\cancel{36}\,3}} = = \frac{43}{63}$ Ans

$\rho_{xy} = \frac{\sigma_{xy}}{\sigma_x \sigma_y} = E(x,y) - E(x) \cdot E(y)$

$\Rightarrow \int_{0}^{1} \int_{0}^{x} xy \; 8xy \; dx\,dy$

$= 8x^2y^2 \, dx\,dy \qquad = \frac{8}{3 \cdot 6} x^6$

$= \left. \frac{8x^2y^3}{3} \right|_{0}^{x} \qquad \frac{8}{18} = \frac{4}{9}$

$\frac{8x^2 x^3}{3} = \frac{8x^5}{3}.$

classmate

$\frac{x^3}{3}\Big|_{1/6}^{1}$

$)$

$\underline{x^2}$

$= \frac{4}{9}$

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$$E(x) = \int_0^1 x \cdot f_x(x)\,dx = 4/5$$

$$E(y) = \int_0^1 8xy\,dx = 4x^2y\Big|_0^1$$

$$\cancel{\int_0^1 y \cdot 4y\,dy} = \cancel{4y^2\,dy} = \frac{\cancel{4y^3}}{3}\Big|_0^1 \quad 4y.$$

$$\Rightarrow \frac{4}{3}$$

$$\int \cancel{8xy} \cdot y(f_{x,y})$$

$$\cancel{f_y = \int_0^x 8xy\,dy} = \frac{8y^2}{2}x\Big|_0^x$$

$$\Rightarrow \cancel{4x[}$$

$$f_y = \int_y^1 8xy\,dx = 4x^2y\Big|_y^1$$

$$4y[1-y^2]$$

$$E(y) = \int_0^1 y \cdot 4y[1-y^2]\,dy$$

$$\int 4y^2 - 4y^4 = \frac{4}{3} - \frac{4}{5} = \frac{20-12}{15} = 8/15$$

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$\frac{4}{9} - \frac{4}{5} \cdot \frac{8}{15} = \frac{20-32}{45}$

$3 | 9, 15 \times 5$
$3, 5\ 5$

$= \frac{-12}{45}$

$\frac{4}{9} - \frac{32}{225} = \frac{100 - 32 \times 3}{225}$

$= \frac{4}{225}$ ~~Ans~~

$\rho \Rightarrow \dfrac{\cancel{\frac{4}{225}}}{\sqrt{\cancel{\frac{4}{5} \times \frac{8}{15}}}} = \text{var} = E(x^2) - (E(x))^2$

$= 0.4923$

(13) $f(x,y) = \frac{1}{4}(1+xy) \quad |x|<1, \ |y|<1$

$P(2x<y)$,

$\int_{-1}^{1}\int_{0}^{y/2} \frac{1}{4}(1+xy)\,dx\,dy$

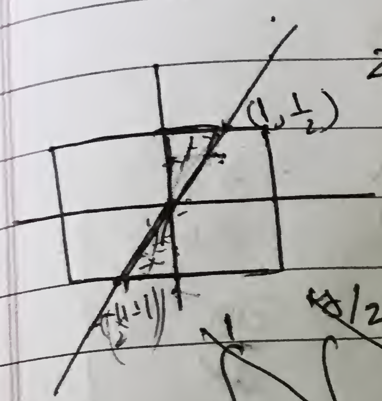
$\Rightarrow \int_{-1}^{1}\int^{y/2} \frac{1}{4}\left(x + \frac{x^2}{2}y\right)\Big|\, dy \qquad \frac{1}{4}\left[\frac{1}{4} + \frac{1}{32} - \right.$

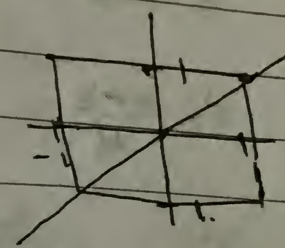
$\frac{1}{4}\int_{-1}^{1}\left(\frac{y}{2} + \frac{y^3}{8}\right) dy$

$\Rightarrow \frac{1}{4}\int_{-1}^{1} y \quad \frac{1}{4}\left(\frac{y^2}{4} + \frac{y^4}{32}\right)_{-1}^{1}$

$$\int_{-1}^{1}\int_{-1}^{y/2} \frac{1}{4}\underline{(1+xy)}\,dx\,dy$$

$2x < y$





$\frac{1}{2}\times\frac{1}{2}\cdot 1$

$$\cancel{\int_{-1}^{1}\int_{0}^{y/2}}$$

$$\int_{0}^{1}\int_{0}^{y/2} \frac{1}{4}(1+xy)\,dx\,dy$$

$$\Rightarrow \int_0^1 dy \left[\frac{x}{4} + \frac{x^2}{8}y\right]\Bigg|_0^{y/2}$$

$$= \int_0^1 dy \cdot \left[\frac{y}{8} + \frac{y^3}{32}\right]$$

$$\Rightarrow \int_0^1 \left(\frac{y}{8} + \frac{y^3}{32}\right) dy = \int_0^1 \frac{y^2}{16} + \frac{y^4}{32\cdot 4}$$

$$\Rightarrow \left(\frac{1}{16} + \frac{1}{32\cdot 4}\right) \times 2$$

$$\Rightarrow \frac{1}{8} + \frac{1}{64} = \frac{8+1}{64}$$

$$= 9/64$$

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Q $f(x,y) = \frac{1}{4} e^{-(x+y)/2}$ $\quad x>0, \; y>0$

$U = \frac{x-y}{4}$ $\qquad V = \frac{x+y}{4}$

$\cancel{\begin{matrix} \frac{\partial U}{\partial x} & \frac{\partial V}{\partial y} \\ \frac{\partial U}{\partial y} & \partial V \end{matrix}}$ $\qquad \begin{vmatrix} \frac{\partial x}{\partial U} & \frac{\partial x}{\partial V} \\ \frac{\partial y}{\partial U} & \frac{\partial y}{\partial V} \end{vmatrix}$

$4U = x-y$

$4V = x+y$ $\qquad \begin{vmatrix} 2 & 2 \\ -2 & 2 \end{vmatrix}$

$2(U+V) = x$

$2(-U+V) = y$

$2(V-U) = y$ $\qquad -\infty < U < \infty$, $\; -\infty < V < \infty$

$= 4+4 = 8$

$V>0$

$8 \cdot \frac{1}{4} e^{-\frac{4V}{2}} = 2e^{-2V}$

PMF $2e^{-2V}$ $(-\infty < V < \infty)$

$f_U \cdot \int_{-\infty}^{\infty} 2e^{-2V} \cdot dV$ $\qquad e^{-2V} \Big|_{-\infty}^{-U}$

$\int_{-\infty}^{-U} 2e^{-2V} dV$

$U+V>0$

$U > -V$ $\quad x>0$ ①

$V>U$ $\qquad V<-U$

$V>U$

$\Rightarrow +\frac{\cancel{2}^{\,-1} e^{-2}}{-2} \Big|_{-\infty}^{-U}$ $\qquad e^{2U} \quad e^{-\infty}$

$-e^{-2v}\Big|_{-\infty}^{-u}$

$$\int_{-\infty}^{-u} 2e^{-2v} dv$$

$$\int_{u}^{\infty} 2e^{-2v} dv$$

$\left[-e^{-2v}\right]_{-\infty}^{-u}$ $-e^{2u}$ $-\frac{1}{e^{(2-\infty)}}$ ∞

$v > -v$ —

$v > u$ - $v < 0$ $u > 0$

$v > 0$ $u < 0$

$f_x(x) = e^{-x} \quad x \geq 0$ $\quad$ $f_y(y) = 2e^{-2y} \quad y \geq 0$

$\quad\quad\quad\; 0 \quad x \leq 0$ $\quad\quad\quad\quad = 0 \quad y \leq 0$

$U = X + Y$. $\quad U - Y = X$

$V = Y$ $\quad\quad U \geq 0$

$V \geq 0 \quad x \geq 0 \quad U - V \geq 0$

$U \geq V$

$U =$

$V \geq 0$

$U \geq V \quad 0 \leq V \leq U$

$U \geq 0$

Q $f(x,y) = 2x e^{-y} \quad 0<x<1,\ y>0$

$= 0$

$X+Y$

$\cancel{X+Y=U}$ $\quad \cancel{X=U-V}$ $\quad X=U$

$\cancel{Y=V}$ $\quad \cancel{V\geq 0}$ $\quad X+Y=V$

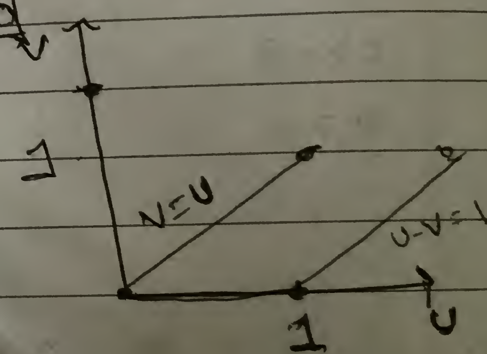
$Y = V-U \qquad 0<U<1$

$V > U$

$\boxed{V>0}$

$0 < U-V < 1$

$1 < U < 2$



Assignment-4

$f_X(x) = \frac{2}{9}(x+1) \qquad -1<x<2.$

$Y = x^2$

$x = \sqrt{y} \qquad 0<x<2$

$\left|\frac{dx}{dy}\right| = \frac{1}{2\sqrt{y}}$

$\frac{1}{2\sqrt{y}} \cdot \frac{2}{9}(\sqrt{y}+1) \qquad 0<y<4.$

when $-1 < x < 0$.

$\frac{dy}{dx} = \frac{-1}{2\sqrt{y}}$ $\quad x = -\sqrt{y}$

$\Rightarrow \frac{-1}{2\sqrt{y}} \cdot \frac{2}{9}(-\sqrt{y}+1)$

$\Rightarrow \boxed{\frac{-1}{9\sqrt{y}}}(-\sqrt{y}+1) = \frac{1}{9} - \frac{1}{9\sqrt{y}}$

$= \frac{1}{9} - \frac{1}{9\sqrt{y}}$ $\quad 0 < y < 1$

$\Rightarrow 2/9$

we have to divide it into Parts one is symmetric & other is increasing or decreasing

$= \frac{1}{2\sqrt{y}}\left[\frac{2}{9}(\sqrt{y}+1) + (-\sqrt{y}+1)\right.$

$\Rightarrow \frac{2}{9 \times 2\sqrt{y}} \times 4 = \frac{2}{9\sqrt{y}}$

For $-~~1 < x < 1~~$. $0 < y < 1$

$\frac{2}{9\sqrt{y}}$

$~~1 < y < 4~~$.

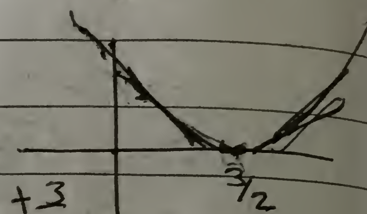
$\frac{1}{9\sqrt{y}}(1+\sqrt{y})$ $\quad 1 < y < 4$

$f(-y)+f(y)$ $\quad x = \frac{3}{2} - \sqrt{y}$

$\sqrt{y} = x - 3/2$

Q2 $\quad f_X(x) = \begin{cases} x/2 & 0 \le x < 1 \\ 1/2 & 1 < x \le 2 \\ (3-x)/2 & 2 < x \le 3 \end{cases}$

$\sqrt{y} = x - 3/2$



$$Y = \left(x - \frac{3}{2}\right)^2$$

$$x - \frac{3}{2} = \pm\sqrt{y} \qquad x = +\sqrt{y} + \frac{3}{2}$$

$$x = -\sqrt{y} + 3/2$$

$$\sqrt{y} \qquad x = \frac{3}{2} + \sqrt{y}$$

$$\left|\frac{dx}{dy}\right| = \frac{1}{2\sqrt{y}} = \frac{1}{2}\left(\frac{3}{2} + \sqrt{y}\right) \times \frac{1}{2\sqrt{y}}$$

$$\Rightarrow \frac{1}{4\sqrt{y}}\left(\frac{3}{2} + \sqrt{y}\right) \quad \frac{1}{4} < y < \frac{9}{4}$$

for $1 < x \le 2$

$$\Rightarrow \frac{1}{2\sqrt{y}}\left(\cancel{\frac{3}{2} + \sqrt{y}} + \cancel{\frac{3}{2} - \sqrt{y}}\right) \quad \frac{1}{2} + \frac{1}{2}$$

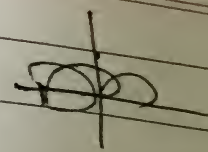
$$\frac{1}{2\sqrt{y}} \Rightarrow \quad 0 < y < \frac{1}{4}$$

for $2 < x \le 3$

$$\frac{1}{4\sqrt{y}}\left(3 - \frac{3}{2} - \sqrt{y}\right) = \frac{1}{4\sqrt{y}}\left(\frac{3}{2} - \sqrt{y}\right) \quad \frac{1}{4} < y < \frac{9}{4}$$

$$= \frac{3}{8\sqrt{y}} - \frac{1}{4}$$

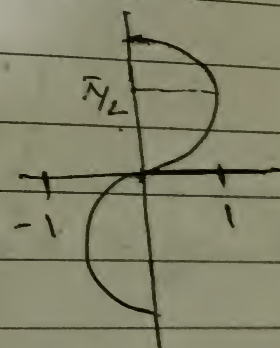
Q3 $f_X(x) = \begin{cases} 2x/\pi^2 & 0 < x < \pi \\ 0 & \end{cases}$



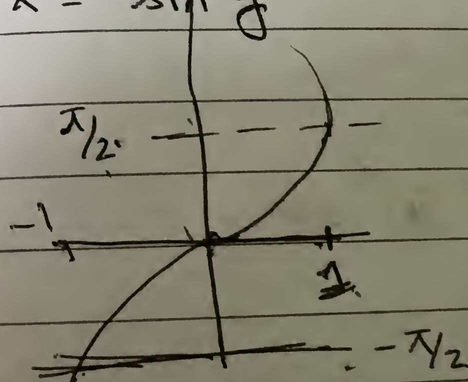
$y = \sin x$

Q. $x = \sin^{-1} y$ $\boxed{-\pi/2 < y < \pi/2}$

$\Rightarrow \dfrac{d \sin^{-1} y}{dy} = \dfrac{1}{\sqrt{1-y^2}}$



$x = \sin^{-1} y$



$-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$

$\boxed{0 < x < \pi}$

$0 < y < \pi/2$

$$\Rightarrow \boxed{\frac{1}{\sqrt{1-y^2}} \; \frac{2 \sin^{-1} y}{\pi^2}}$$

Q4 $X \sim Bin(n, p)$ ${}^nC_r\, p^r q^{n-r}$

$Y_1 = 3X + 4$

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Q4 $Y_1 = 3x + 4$ $\quad$ $Bin(n, p)$

$${}^nC_x \, p^{x} q^{n-x}$$

$$\frac{y-4}{3} = x \qquad {}^nC_{\frac{y-4}{3}} \, p^{\frac{y-4}{3}} q^{n-\frac{y+4}{3}}$$

$$\Rightarrow \binom{n}{\frac{y-4}{3}} p^{\frac{y-4}{3}} (1-p)^{n-\frac{y+4}{3}}$$

c $Y_3 = x^2 + 2$

$$e^{-1/2}$$

$$16C_0 \cdot (p)^{16}$$

4 $\quad$ 9

$$\frac{e^{-2} \lambda^n}{n!} = e^{-2} \lambda^x \quad \min(x, 10)$$

$$P(X=1) = \frac{e^{-2} 2^1}{1}$$

$$P(X=2) = \frac{e^{-2} 2^2}{2!}$$

Q6

$x = 3$

$$\frac{\frac{3}{4} e^{-2} \cdot \cancel{2^0}}{\frac{3}{4} e^{-2} + \frac{1}{4} e^{-3}}$$

Q5

Q7

Q9

$f(x) =$

E

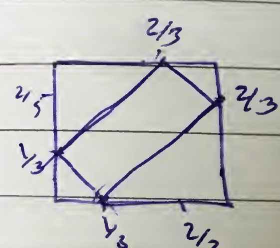
Q5 $\lambda = 2$ $= \dfrac{e^{-2t}(2t)^{n}}{n!}$

$\Rightarrow \dfrac{e^{-2}\cdot 2^{n}}{n!}$ since $t = 1$

Q7 $\dfrac{1}{\lambda} = 300$ $\lambda = 1/300$ $P(X \le 4)$

$\Rightarrow e^{-6} + \dfrac{e^{-6}6}{1} + \dfrac{e^{-6}\cdot 6^{2}}{2!} + \dfrac{e^{-6}6^{3}}{3!}$

$\Rightarrow \dfrac{0.2}{100}\lambda$



$\Rightarrow \cancel{2 \times \frac{1}{2} \times \frac{1}{3} \times \frac{1}{3}}$

$1 - \left(\cancel{2} \times \frac{1}{\cancel{3}} \cdot \frac{1}{3}\right)$

$\dfrac{1}{\cancel{2}} \cdot \dfrac{\cancel{2}}{3} \cdot \dfrac{2}{3} \times 2 = 1 - \dfrac{4}{9} = 5/9$

Q9 $f_x = 4/5c$ $\left(\frac{3}{4}c < x < 2c\right)$

↳ const

$f(x) = \dfrac{1}{b-a}$

~~$f(c) = 7/5c$~~

$E(x)$ $\cancel{\dfrac{2c + \frac{3}{4}c}{2}}$

$\Rightarrow \cancel{\dfrac{11c}{8}}$

$f(c) = 4/5c$

Let Kc be bid

Profit $= Kc - c$

$= c(K-1)$

$E(P) = c(K-1)f(c)$

$$\int_{Kc}^{2c} \cancel{c}(K-1)\cdot\frac{4}{5\cancel{c}}\,dc$$

$$(K-1)4\left[2c-Kc\right]$$

$$4(K-1)c(2-K) = 4c(2-K)(K-1)$$

$$\Rightarrow 4c(2K-2-K^2+K)$$

$$= 4c(-K^2+3K-2)$$

$$\Rightarrow 4c(-2K+3) = 0$$

$$K = 3/2$$

Q10 $\cancel{\emptyset}$ $f(x) = \lambda e^{-\lambda x}$

$\frac{1}{\lambda} = 50$

$\lambda = \frac{1}{50}$

$1-(P(X=0)+P(X=1))$

$1-\frac{1}{50}e^{0}+\frac{1}{50}e^{-50}$

Probability of bulb which will glow more than 100 hr

$$P(T>t) = e^{-1/50\times 100} = e^{-2} = p$$

$$1-{}^{10}C_0\, p^0(1-e^{-2})^{10}+{}^{10}C_1\, e^{-2}(1-e^{-2})^{9}$$

$$1-(1-e^{-2})^{10}-10e^{-2}(1-e^{-2}) = 0.400$$

Q11

Q12

Q13

Q11

A | B
$\frac{1}{\lambda_1} = 5$ | $\frac{1}{\lambda_2} = 2$
$\lambda_1 = 1/5$ | $\lambda_2 = 1/2$

$P(B) = \frac{3}{4}$ $P(A) = \frac{1}{4}$

$$\frac{3}{4} e^{-5/2} + \frac{1}{4} e^{-5/5} = \frac{3}{4} e^{-5/2} + \frac{1}{4} e^{-1}$$

Q12 $P(T > t)$

$$R(t) = \prod_{i=1}^{n} R_i(t)$$

$$\Rightarrow \cancel{e^{-\lambda t_1 t}} = e^{-t \sum_{i=1}^{n} \lambda_i}$$

$$\Rightarrow 1 - e^{-t \sum_{i=1}^{n} \lambda_i}$$

Q13

$$\frac{\gamma}{\lambda} = 20 \qquad \sqrt{\gamma} \qquad \frac{\sqrt{\gamma}}{\lambda} = 10$$

$$\Rightarrow \sqrt{\gamma} = 2 \qquad \lambda = 0.2$$

$$\gamma = 4$$

$$P(T_\gamma > t) = \sum_{j=0}^{4-1} \frac{e^{-\lambda t} (\lambda t)^j}{j!}$$

$$\Rightarrow \sum_{j=0}^{3} \frac{e^{-3} (3)^j}{j!}$$

within 15 days

$$= 1 - P(T_r > 15)$$

9

400

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Q14 $f_n(x) = \alpha\beta x^{\beta-1} e^{-\alpha n^\beta}$

$$P \quad \frac{1-e^{-\alpha\cdot 100^2}}{e^{-\alpha\cdot 90^2}} = 0.15$$

$$\frac{1-e^{-\alpha 1000}}{e^{-8100\alpha}} = 0.15$$

$$1 - e^{10000\alpha} = 0.15\, e^{-8100\alpha}$$

$$1 = 0.15 e^{-8100\alpha} + e^{10090\alpha}$$

$$\Rightarrow \frac{1}{0.15} e^{8100\alpha} + e^{18100\alpha} = 1$$

Q15

$P(90 < n < 100)$

$$\Rightarrow \frac{e^{-\alpha\cdot 90^2} - e^{-\alpha 100^2}}{e^{-\alpha 90^2}}$$

$$\Rightarrow \frac{e^{-8100\alpha} - e^{-10000\alpha}}{e^{-8100\alpha}}$$

$$\Rightarrow 1 - e^{-1900\alpha} = 0.15$$

$0.85 = e^{-1900\alpha}$ $\quad \alpha = \boxed{0.8555}$

Q15

$\sigma = \frac{30}{2.9}$

Q.15

$P(x < 45) = 0.10$

$P(45 < x < 60)$

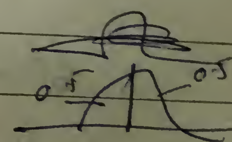
$P(60 < x < 75)$

$P(x > 75) = 0.5$

$P\left(-\infty < x < \frac{45-\mu}{\sigma}\right) = 0.10$

$P\left(\frac{75-\mu}{\sigma} < X < \infty\right) = 0.05$

$\therefore P\left(0 < z < \frac{45-\mu}{\sigma}\right) = 0.40$



$\phi\left(\frac{45-\mu}{\sigma}\right) - \phi(0) = 0.4$

$\phi\left(\frac{45-\mu}{\sigma}\right) = 0.1$ & $\phi\left(\frac{75-\mu}{\sigma}\right) = 0.5+0.5-0.05 = 0.45$

$\frac{75-\mu}{\sigma} = 1.66$

$\frac{45-\mu}{\sigma} = -1.28$

$\mu = 1.28\sigma + 45$

$\frac{\mu-45}{\sigma} = 1.28$

$\frac{75-\mu}{\sigma} = 1.66$

$75 - 1.28\sigma - 45 = 1.66\sigma$

$30 = 1.66\sigma + 1.28\sigma$

$\sigma = \frac{30}{2.94} = 10.20$

$30 = 1.94\sigma$

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$\mu = 58.06$

$\phi(45 < x < 60)$

$\Rightarrow \phi\left(\frac{45 - 58.06}{10.20} < x < \frac{60 - 58.06}{10.20}\right)$

$\Rightarrow \phi(-1.28)$

$\phi(.19) - \phi(-1.28)$

$0.5753 - .1003 = 0.475$

$\phi(\cancel{60} < x < 75)$

$\phi(1.66) - \phi(.19)$

$0.9505 - 0.5753 = 0.3752$

Q6 $\mu = 3$ $\sigma = 0.005$

$d_1 = 3 - 0.01 = 2.99$

$d_2 = 3 + 0.01 = 3.01$

$\phi\left(\frac{2.99 - 3}{0.005} < x < \frac{3.01 - 3}{0.005}\right)$

$\Rightarrow \phi((-2) < x < 2)$

$\phi(2) - \phi(-2)$

$\cancel{2\phi(2) = 1 -}$

$0.9772 - 0.0228$

$\Rightarrow .9544$

Ans = $1 - .9544$

0017

Sol $\mu = 0.9$, $\sigma = 0.003$

$$P\left(\frac{.9 - 0.005}{0.003} < x < \frac{.9 + 0.005}{0.003}\right)$$

$$P\left(\frac{.895 - 0.9}{0.003} < x < \frac{.905 - 0.9}{0.003}\right)$$

$$\therefore P(-1.66 < x < 1.66) = \phi(1.66) - \phi(-1.66)$$

$$\Rightarrow 2\phi(1.66) - 1$$

$$= 0.901$$

% of defective $= 1 - 0.901$

$= 0.099$

$$P\left(\frac{0.895}{\sigma} < x < \frac{0.905}{\sigma}\right) = 0.99$$

$$\Rightarrow \cancel{\phi\left(\frac{0.905}{\sigma}\right) - \phi\left(\frac{0.895}{\sigma}\right) = \phi(2.33)}$$

$$\Rightarrow \cancel{\frac{0.10}{\sigma} = 2.33}$$

$$\cancel{\sigma = \frac{0.10}{2.33}}$$

$$\phi\left(\frac{0.905 - 0.9}{\sigma}\right) - \phi\left(\frac{0.895 - 0.9}{\sigma}\right) = \phi(2.33)$$

$$\Rightarrow 2\phi(0.90) - 1 = \phi(2.33)$$

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Q18 $\mu = 2$ $\sigma = 10$ $P(A$

$\cancel{P\left(\frac{x+\mu}{\sigma}\right) = 0.05}$

$\cancel{\phi\left(\frac{x+2}{10}\right) = 0}$

$\phi\left(\frac{200-x}{10}\right) = 0.95$

$\frac{200-x}{10} = 1.66$

$200 - x = 16.6$

$x = 183.4$

$\cancel{\frac{x-\mu}{\sigma} = -1.28}$ $P(z > 0.1)$

$\cancel{\frac{x-200}{10} = -1.28}$

$\cancel{x = -12.8+200}$

$=$

Q19 $\mu = 74$ $\sigma^2 = 62.41$

$\sigma = 7.9$

$\boxed{0 < x < 4}$

$P\left[\frac{\mu - n}{\sigma}\right]$ $P\left(z < \frac{\mu - n}{\sigma}\right)$

$f_r(t) = \frac{\lambda^{r} t^{r} e^{-\lambda}}{\Gamma r}$ → To find Probability of this we need to integrate it.

$$\int \frac{\lambda^{r} t^{r} e^{-\lambda}}{\Gamma r}$$

$$z(t) = 0.027 + 0.00025(t-40)^2$$

$$R = e^{-\alpha n^{\beta}} \quad H = \alpha\beta e^{-\alpha n^{\beta}}$$

$$R(t) = e^{-\int_{40}^{t} z(s)\,ds}$$

$$\alpha = 0.006 \ \& \ \beta = 0.5$$

$$R(t) = e^{-\alpha t^{\beta}}$$

$$e^{-\lambda t} \Rightarrow e^{-\frac{1}{25000}t}$$

$$\Rightarrow e^{-\alpha t^{\beta}} \cdot e^{-\frac{t}{\lambda}} \qquad e^{-\alpha t}(1-R t)+\alpha t)$$

P(T<2000)

A
B

$$e^{-\alpha t}(1-R(t)] + (1-e^{-\alpha t})R(t)$$

$$\& \ e^{-\alpha t}R(t)$$

$$e^{-\alpha t} - \cancel{e^{-\alpha t}R(t)} + R(t) - e^{-\alpha t}R(t)$$

$$\cancel{+e^{-\alpha t}R(t)}$$

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Q8 $P(T<t) = 1-e^{-\lambda t}$ $\int_0^4$

$= 1-e^{-(\lambda)t}$

$\boxed{P(n>2)}$ $\mu = 160$; σ

$= 4$ $P(120 < x < 200) = 0.80$

$\Rightarrow$ $\cancel{\phi\left(\frac{200-\mu}{\sigma}\right)}$

$P\left(-\frac{40}{\sigma} < x < \frac{40}{\sigma}\right) = 0.80$ $\phi(n)+\phi(-n)=1$

$\Rightarrow \phi\left(\frac{40}{\sigma}\right) - \phi\left(-\frac{40}{\sigma}\right) = 0.80$ σ

$\Rightarrow$ $\cancel{1-2\phi\left(\frac{40}{\sigma}\right)=0.80}$

$2\phi\left(\frac{40}{\sigma}\right)-1 = 0.80$

$P(x \geq 170) / (x \geq 140)$ $\Rightarrow \dfrac{P\left(\frac{160-170}{\sigma}\right)}{P\left(\frac{140-160}{\sigma}\right)}$

$\Rightarrow$ $\Rightarrow$ $\frac{10}{30}$

Q21

$c_0 \cdot P(6<x<8) - c_1 P(x<6) - c_2 P(x>8)$

$\Rightarrow c_0\left(\frac{6-\mu}{\sigma} < x < \frac{8-\mu}{\sigma}\right) - c_1\left(\frac{6-\mu}{\sigma}\right) - c_2\left(1-\phi\left(\frac{8-\mu}{\sigma}\right)\right)$

$c_0\left(\phi(8-\mu) - \phi(6-\mu)\right) - c_1\phi(6-\mu)$

$- c_2 + c_2\phi(8-\mu)$

0.80

$\phi(n)+\phi(-n)=1$

$P(X>8)$

$(1-\phi\left(\frac{8-\mu}{\sigma}\right))$

$c_2\,\phi(8-\mu)$

$c_0\left[\phi(8-\mu)-\phi(6-\mu)\right]-c_1\phi(6-\mu)$
$+c_2\phi(8-\mu)$

$c_0\left(e^{-\frac{(8-\mu)^2}{2}}-e^{-\frac{(6-\mu)^2}{2}}\right)-c_1 e^{-\frac{(6-\mu)^2}{2}}$
$+c_2\,e^{-\frac{(8-\mu)^2}{2}}=0$

$\phi(8-\mu)(c_0+c_2)-\phi(6-\mu)(c_0+c_1)=0$

$\Rightarrow \cancel{c_0 e^{-\left(\frac{8-\mu}{2}\right)^2}\left[1-e^{-[(6-\mu)^2-(8-\mu)^2]}\right]}$

$\Rightarrow c_1 e^{-(6-\mu)^2}+c_2 e^{-(8-\mu)^2}$

$e^{-\frac{(8-\mu)^2}{2}}\left[c_0+c_2\right)=\left(c_1+c_0\right)e^{-\frac{(6-\mu)^2}{2}}$

$$\frac{e^{-\left(\frac{8-\mu}{2}\right)^2}}{e^{-\left(\frac{6-\mu}{2}\right)^2}}=\frac{c_1+c_0}{c_0+c_2}$$

$\Rightarrow$ Taking ln

$\Rightarrow -2\left(\frac{8-\mu}{2}\right)+2\left(\frac{6-\mu}{2}\right)=\frac{c_1+c_0}{c_0+c_2}$

$-8+\mu+6-\mu$

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$$\ln e^{-\left(\frac{8-y}{\sqrt{2}}\right)^2} - \ln e^{-\left(\frac{6-y}{\sqrt{2}}\right)^2}$$

$$\Rightarrow -2\left(\frac{8-y}{\sqrt{2}}\right) + 2\left(\frac{6-y}{\sqrt{2}}\right)$$

$$\Rightarrow \frac{-16+2y}{\sqrt{2}} + \frac{12-\cancel{2y}}{\sqrt{2}}$$

$$\Rightarrow \frac{-4}{\sqrt{2}}$$

$$\ln(A/B) = \ln A - \ln B$$

$$= \ln e^{-\left(\frac{8-y}{\sqrt{2}}\right)^2} - \ln e^{-\left(\frac{6-y}{\sqrt{2}}\right)^2}$$

$$\Rightarrow -\left(\frac{8-y}{\sqrt{2}}\right)^2 + \left(\frac{6-y}{\sqrt{2}}\right)^2$$

$$\Rightarrow \frac{-(8^2+y^2-16y) + (36+y^2-12y)}{2}$$

$$\Rightarrow \frac{-64-\cancel{y^2}+16y+36+\cancel{y^2}-12y}{}$$

$$\begin{array}{r} -64 \\ 36 \\ \hline 28 \end{array}$$

$$\Rightarrow \frac{-28+4y}{2} = -7+y$$

$$-7+y = -14+2y =$$

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$$\left(\frac{6-\mu}{\sqrt{2}}\right)^2 - \left(\frac{8-\mu}{\sqrt{2}}\right)^2$$

$$\frac{(6-\mu)^2-(8-\mu)^2}{2} = \frac{36+\mu^2-12\mu-(64+\mu^2-16\mu)}{2}$$

$$\Rightarrow \frac{36+\cancel{\mu^2}-12\mu-64-\cancel{\mu^2}+16\mu}{2}$$

$$\Rightarrow \frac{-28+4\mu}{2} =$$

$$-14+2\mu = \frac{c_0+c_1}{c_2+c_0}$$

$$P'\left(z \le \ln\left(\frac{x-0.8}{0.1}\right)\right)$$

$$1-\phi\ \ln\left(\frac{2.7-0.8}{0.1}\right) \qquad 1-P\left(z \le \ln(2.7)\right)$$

$$\Rightarrow 1-\ln\left(\frac{1.9}{0.1}\right)$$

$$1-\ln 19 \qquad \Rightarrow 1-\phi\left(\frac{\ln(2.7)-\mu}{\sigma}\right)$$

$$1-.9984 \qquad \Rightarrow 1-\phi\left(\frac{0.993-0.8}{0.1}\right)$$

y is 0

$x = e^y$

$$\cancel{1-\phi\left(\frac{0.193}{0.1}\right)}$$

$$1-\phi(0.193)$$

$$1-0.9732$$

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$\frac{z-\mu}{\sigma}$ $\frac{z+\mu}{\sigma}$ $Y = \ln x$ $\frac{z-\mu}{\sigma}$ $\frac{\mu-z}{\sigma}$

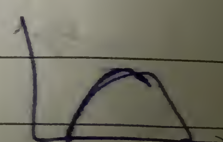
$\frac{0.8-2.71}{0.1}$

$\cancel{P(Y < \ln x < z) = 0.95}$

$P\left(\frac{\mu-y}{\sigma} < Z < \frac{\mu+y}{\sigma}\right) = 0.95$

$= 2\phi\left(\frac{\ln(z)-\mu}{\sigma}\right) - 1 = 0.95$

$= 2\phi\left(\frac{\ln(z)-\mu}{\sigma}\right) = 1.95$



$\phi\left(\frac{\ln(z)-\mu}{\sigma}\right) = 1.95/2$

$\phi\left(\frac{\ln(z)-0.8}{0.1}\right) = 1.95/2$

$\frac{\ln(z)-0.8}{0.1} = 1.97$

$\ln z - 0.8 = 0.197$ $\qquad$ $\ln z = 0.997$

$z = 2.71$ Ans

$\cancel{\frac{2.71-0.8}{0.1} \Rightarrow \frac{1.91}{0.1}}$ $\Rightarrow$ $\frac{0.8-0.99}{0.1}$

$\frac{0.8-\ln z}{0.1} = 1.97$

$0.603 = \ln(z)$

Assignt-6

Q1) $U = |u|$ & $V = y^2$

$0 \le U \le 1$ $\quad 1 \le V \le 4$

$P(U=0 \mid V=1) = 1/12$

$P(U=0 \mid V=4) = 1/12$

$P(U=1 \mid V=1) = 1/6 + 1/6 = 1/3$

$P(U=1 \mid V=4) = 1/6 + 1/6 + 1/12 + \frac{1}{12}$

$\Rightarrow \frac{1}{3} + \frac{1}{6} = \frac{2+1}{6} = \frac{1}{2}$

V \ U	0	1
1	1/12	1/3
4	1/12	1/2

Q2) $z^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$

$\Rightarrow \dfrac{x^2 + y^2}{}$ $\qquad \dfrac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$

$x = x_1 - x_2$

$y = y_1 - y_2$

$x_1 - x_2 \sim N(0,1)$

$y_1 - y_2 \sim N(0,1)$

Q3.

$X_1 = \lambda e^{-\lambda x_1}$ $\quad X_2 = \lambda e^{-\lambda x_2}$

$\cancel{Y_1 = X_1/X_2 = \frac{\lambda e^{-\lambda x_1}}{\lambda e^{-\lambda x_2}}}$

$\cancel{Y = e^{-\lambda(x_2 - x_1)}}$

$Y_1 = X_1/X_2$

$Y_2 = X_1 + X_2$

$Y_2 = Y_1 X_2 + X_2$

$X_2 = \frac{Y_2}{1+Y_1}$

$\therefore X_1 = \frac{Y_1 Y_2}{1+Y_1}$

$$\begin{vmatrix} \frac{dx_1}{dy_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{dx_2}{dy_1} & \frac{dx_2}{dy_2} \end{vmatrix} = \begin{vmatrix} \frac{y_2}{1+y_1} & \frac{y_1}{1+y_1} \\ \frac{-y_2}{(1+y_1)^2} & \frac{1}{1+y_1} \end{vmatrix}$$

$$= \cancel{\frac{y_2}{(1+y_1)^2} + \frac{y_1 y_2}{(1+y_1)^3}}$$

$$\Rightarrow \begin{vmatrix} \frac{-y_2 y_1}{(1+y_1)^2} + \frac{y_2}{(1+y_1)} & \frac{y_1}{1+y_1} \\ \frac{-y_2}{(1+y_1)^2} & \frac{1}{1+y_1} \end{vmatrix}$$

$$\Rightarrow \cancel{\frac{-y_2 y_1}{(1+y_1)^3}} + \frac{y_2}{(1+y_1)^2} + \cancel{\frac{y_1 y_2}{(1+y_1)^3}}$$

$$\Rightarrow \frac{y_2}{(1+y_1)^2}$$

$$f(x_1, x_2) = \lambda^2 e^{-\lambda(x_1+x_2)} \cdot \left| \frac{y_2}{(1+y_1)^2} \right|$$

$$f(x_1, x_2) = \frac{\lambda^2 e^{-\lambda(x_1+x_2)} \cdot y_2}{(1+y_1)^2}$$

$$f(y_1, y_2) = \frac{\lambda^2 e^{-\lambda y_2} y_2}{(1+y_1)^2} \text{ — Joint}$$

$$f_{y_1} \Rightarrow \int_0^\infty \frac{\lambda^2 e^{-\lambda y_2} y_2}{(1+y_1)^2} \cdot dy_2$$

$$\Rightarrow \frac{\lambda^2}{1+y_1^2} \int_0^\infty y_2 e^{-\lambda y_2} \, dy_2$$

$$= \frac{\lambda^2}{1+y_1^2} \left[y_2 \frac{e^{-\lambda y_2}}{\lambda} - \int -\frac{e^{-\lambda y_2}}{\lambda} \right]$$

$$\frac{\lambda^2}{1+y_1^2} \left[\frac{y_2 e^{-\lambda y_2}}{\lambda} - \frac{e^{-\lambda y_2}}{\lambda^2} \right|_0^\infty$$

$$\frac{\lambda^2}{1+y_1^2} \left[0 - \left(0 - \frac{1}{\lambda^2}\right) \right) = \frac{1}{(1+y_1)^2}$$

x_1

Q) $Y_1 = x_1^2 + x_2^2 \qquad Y_2 = x_1/x_2 \qquad Y_1 > 0$

$-\infty < Y_2 < \infty$

F $\quad \Phi = \int_0^\infty \frac{1}{\sqrt{2\pi}} e^{-z^2/2}$

$\phi(x_1) = \frac{1}{\sqrt{2\pi}} e^{-x_1^2/2} \qquad \phi(x_2) = \frac{1}{\sqrt{2\pi}} e^{-x_2^2/2}$

$\Rightarrow \quad Y_1 = \cancel{y}_2^2 x_2^2 + x_2^2$

$y_1 = x_2^2(1+y_2^2)$

$x_2^2 = y_1 / 1+y_2^2 \qquad \therefore x_2 = \left(\frac{y_1}{1+y_2^2}\right)^{1/2}$

$Y_1 = x_1^2 + \frac{y_1}{1+y_2^2}$

$y_1 - \frac{y_1}{1+y_2^2} = x_1^2 \Rightarrow \frac{y_1 + y_1 y_2^2 - y_1}{1+y_2^2} = x_1^2$

$\therefore x_1^2 = \frac{+y_1 y_2^2}{1+y_2^2}$

$\therefore \quad x_1 = \left(\frac{+y_1 y_2^2}{1+y_2^2}\right)^{1/2}$

$x_2 = \left(\frac{y_1}{1+y_2^2}\right)^{1/2}$ $x_1 = \left(\frac{y_1 y_2^2}{1+y_2^2}\right)^{1/2}$

$$\begin{vmatrix} \frac{dx_1}{dy_1} & \frac{dx_1}{dy_2} \\ \frac{dx_2}{dy_1} & \frac{dx_2}{dy_2} \end{vmatrix}$$

$$\Rightarrow \begin{vmatrix} \frac{y_2}{\sqrt{1+y_2^2}} \cdot \frac{1}{2\sqrt{y_1}} & \sqrt{y_1}\left[\frac{1}{2\sqrt{\frac{y_2^2}{1+y_2^2}}} \cdot \left[y_2^2\right.\right. \end{vmatrix}$$

$$J = \frac{-1}{2(1+y_2^2)}$$

$$f(x_1, x_2) = \frac{1}{2\pi} e^{-\frac{y_1}{2}} \times \frac{1}{2(1+y_2^2)}$$

$$\frac{(1+y^2)}{1/2}$$

$x_2 = \sqrt{\frac{y_1}{1+y_2^2}}$ $\quad x_1 = \sqrt{\frac{y_1 y_2^2}{1+y_2^2}}$

$x_2 = -\sqrt{\frac{y_1}{1+y_2^2}}$ $\quad x_1 = -y_2\sqrt{\frac{y_1}{1+y_2^2}}$

$\frac{dx_1}{dy_1}$ $\quad \frac{dx_1}{dy_2}$

$\frac{dx_2}{dy_1}$ $\quad \frac{dx_2}{dy_2}$

$$\frac{dx_1}{dy_1} = \frac{-y_2}{\sqrt{1+y_2^2}} \cdot \frac{1}{2\sqrt{y_1}}$$

$$\Rightarrow \frac{-y_2}{2\sqrt{y_1(1+y_2^2)}}$$

$$\frac{dx_1}{dy_2} = -\sqrt{y_1}\left[y_2 \cdot \frac{(1+y_2^2)^{1/2}}{1/2} \cdot 2y_2 - \frac{1}{\sqrt{1+y_2^2}}\right]$$

$$\Rightarrow -\sqrt{y_1}\left[4y_2^2 (1+y_2^2)^{1/2} - \frac{1}{\sqrt{1+y_2^2}}\right]$$

$$\frac{dx_2}{dy_1} = \frac{-1}{\sqrt{1+y_2^2}} \cdot \frac{1}{2\sqrt{y_1}} = \frac{-1}{2\sqrt{y_1(1+y_2^2)}}$$

$$\frac{dx_2}{dy_2} = -\sqrt{y_1}\,\sqrt{1+y_2^2}$$

$$\frac{dx_1}{dy_2} = -\sqrt{y_1}\left[\frac{y_2 \cdot 2y_2}{(1+y_2^2)^{3/2}} - \frac{1}{\sqrt{1+y_2^2}}\right]$$

$$= -\sqrt{y_1}\left[\frac{2y_2^2}{(1+y_2^2)^{3/2}} - \frac{1}{\sqrt{1+y_2^2}}\right]$$

$$\frac{dx_2}{dy_2} = -\sqrt{y_1}\,(1+y_2^2)^{-3/2}\cdot 2y_2$$

$$\Rightarrow \frac{y_2^2 \cancel{\sqrt{y_1}}}{2\cancel{\sqrt{y_1}}\,(1+y_2^2)^2} = \frac{y_2^2}{2(1+y_2^2)^2}$$

$$\boxed{\frac{-y_2^2}{(1+y_2^2)^2} + \frac{1}{2\sqrt{y_1}\,(1+y_2^2)} \quad \Rightarrow \frac{1}{\cancel{2\sqrt{y_1}\,(1+y_2^2)^2}}}$$

$$\frac{\lambda^r}{\Gamma r}$$

$$J = \frac{y_2^2}{2(1+y_2^2)^2}$$

$$2 \text{ inverse image} = \frac{y_2^2}{(1+y_2^2)^2}$$

Q5/ $f(y_1, y_2) = \frac{1}{2\pi} e^{-\frac{y_1}{2}} \frac{y^2}{(1+y_2^2)^2}$

$f(x\lambda) = \frac{\lambda^r}{\Gamma r} = \frac{\lambda^n}{\Gamma n}$

$y_1 = \frac{x_1}{x_1+x_2}$ $\quad z = \underline{x_1+x_2}$

$y_1 = \frac{x_1}{z} \quad = \quad x_1 = yz$

$x_2 = z(1-y)$

$f_n = \frac{\lambda^r t^{r-1} e^{-\lambda t}}{\Gamma r} = \frac{\lambda^{n_1} x_1^{n_1-1} e^{-x_1\lambda}}{\Gamma n_1}$

$\cancel{f(x_1) = \frac{\lambda^{n_1} t^{n_1-1} e^{-\lambda t}}{\Gamma n_1}}$

$f(x_2) = \frac{\lambda^{n_2} x_2^{n_2-1} e^{-x_2\lambda}}{\Gamma n_2}$

$x_2 = z - zy$

$J = $ $x_1 = yz \quad x_2 = z(1-y)$

$\cancel{z - zy + y^2}$

$z - zy + zy$

$= z$

$\begin{vmatrix} \frac{dx_1}{dy} & \frac{dx_1}{dz} \\ \frac{dx_2}{dy} & \frac{dx_2}{dz} \end{vmatrix} = \begin{vmatrix} z & y \\ -z & (1-y) \end{vmatrix}$

$J = z$

$$= \frac{\lambda^{n_1} \cdot x_1^{n_1-1} e^{-x_1\lambda}}{\Gamma n_1} \cdot \frac{\lambda^{n_2} x_2^{n_2-1} e^{-x_2\lambda}}{\Gamma n_2} \cdot z$$

$$\Rightarrow \frac{\lambda^{(n_1+n_2)} \cdot e^{-(x_1+x_2)\lambda} \cdot x_1^{n_1-1} \cdot x_2^{n_2-1} \cdot z}{\Gamma n_1 \, \Gamma n_2}$$

$$\Rightarrow \frac{\lambda^{(n_1+n_2)} \cdot e^{-z\lambda} \cdot x_1^{n_1-1} \cdot x_2^{n_2-1} \cdot z}{\Gamma n_1 \quad \Gamma n_2}$$

$$\frac{\lambda^{(n_1+n_2)} e^{-z\lambda} (yz)^{n_1-1} \cdot (z(1-y))^{n_2-1} \cdot z}{\Gamma n_1 \quad \Gamma n_2}$$

$$\Rightarrow \lambda^{(n_1+n_2)} \cdot e^{-z\lambda} y^{n_1-1} \cdot z^{n_1-1} \cdot (z(1-y)^{n_2-1}) \cdot z$$

$$\Rightarrow \lambda^{(n_1+n_2)} e^{-z\lambda} y^{n_1-1} z^{n_1} \cdot [z(1-y)]^{n_2-1}$$

Q $f(x) = e^{-x}$

$$y_1 = x_1 + x_2 + x_3$$

$$y_2 = \frac{x_1 + x_2}{y_1} \qquad y_3 = \frac{x_1}{x_1 + x_2}$$

$$x_1 + x_2 = y_2 y_1 \qquad x_1 = y_2 y_1 y_3$$

$$y_1 - y_2 y_1 = x_3 \qquad x_2 = y_2 y_1 - y_2 y_1 y_3$$

$$= y_2 y_1 (1 - y_3)$$

$$\begin{matrix} \frac{dx_1}{dy_1} & \frac{dx_1}{dy_2} & \frac{dx_1}{dy_3} \\ \frac{dx_2}{dy_1} & \frac{dx_2}{dy_2} & \frac{dx_2}{dy_3} \\ \frac{dx_3}{dy_1} & \frac{dx_3}{dy_2} & \frac{dx_3}{dy_3} \end{matrix}$$

$$\Rightarrow \begin{vmatrix} y_2 y_3 & y_1 y_3 & y_2 y_1 \\ y_2 - y_2 y_3 & y_1 - y_1 y_3 & -y_2 y_1 \\ 1 - y_2 & -y_1 & 0 \end{vmatrix}$$

$$\Rightarrow y_2 y_3 - y_2 y_1^2 - y_1 y_3 (y_2 y_1 - y_2^2 y_1)$$

$$+ y_2 y_1 (\cancel{-y_2 y_1} + y_1 y_2 y_3 - y_1 + y_1 y_3 + \cancel{y_2 y_1} - y_1 y_3 y_2$$

$$y_2 y_3 - y_2 y_1^2 - y_1^2 \cancel{y_3} y_2 + y_1^2 y_2^2 y_3$$

$$+ \; y_2 y_1^2 + y_1^2 \cancel{y_2} y_3$$

$$\boxed{y_2 y_3 - y_2 y_1^2 + y_2 y_1^2 + y_1^2 y_2^2 y_3}$$

$$-y^2 \cancel{y_1^2} y_3 - 2y_2 y_1^2 + y_1^2 y^2 \cancel{y_3}$$

$$= -2y_2 y_1^2$$

$$e^{-(x_1+x_2+x_3)}$$

$$f(x,y) \Rightarrow e^{-y_1} \times [-2y_2 y_1^2]$$

$$\Rightarrow 2y_2 y_1^2 e^{-y_1} \quad \text{or} \quad y_2 y_1^2 e^{-y_1}$$

$$\Rightarrow$$

$$f(y_1) = \int_0^1 y_2 y_1^2 e^{-y_1} \, dy_2$$

$$\Rightarrow y_1^2 e^{-y_1} \cdot \frac{y_2^2}{2} \Big|_0^1$$

$$\frac{y_1^2 e^{-y_1}}{2}$$

$$f(y_2) = \int_0^\infty y_1^2 e^{-y_1} \, dy_1$$

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$$\int_0^\infty y_1^2 e^{-y_1} dy_1 \Rightarrow y_1^2(-e^{-y_1}) - \int -e^{-y_1} \cdot 2y_1$$

$$-y_1 e^{-y_1} + 2\int e^{-y_1} y_1 dy_1$$

$$\Rightarrow -y_1 e^{-y_1} - \int -e^{-y_1}$$

$$\Rightarrow -y_1 e^{-y_1} + e^{-y_1}$$

$$-y_1 e^{-y_1} - 2y_1 e^{-y_1} + 2e^{-y_1}$$

$$= -3y_1 e^{-y_1} + 2e^{-y_1} \Big|_0^\infty$$

$$\Rightarrow 0 \; 0 \; \underline{\underline{+2y_2}} = 2y_2$$

Multiply

$$y_1^2 e^{-y_1} \cdot 2y_2 = y_2 y_1^2 e^{-y_1} \text{ independent.}$$

since $\int_0^1 \int_0^\infty f(y_1 y_2) dy_3 = 1$

08

$x^2-2xy+y^2+xy$

$(x-y)^2+xy$

$y_1 = \ln x_1 \sim N(4,1)$

$y_2 = \ln x_2 \sim N(3,1)$ $\quad$ $y_3 = \ln x_3 \sim N(2, 0.5)$

08/ $\cancel{y_1 y_2 y_3 = \ln(x_1}$

$y_1+y_2+y_3 = \ln(x_1 x_2 x_3)$

$\cancel{\ln(w)} = \cancel{2\ln x_1^2}$

$w = e^2\, x_1^2\, x_2^{1.5}\, x_3^{1.28}$

$\ln w = \ln\left(e^2 \cdot x_1^2 \cdot x_2^{1.5}\, x_3^{1.28}\right)$

$\Rightarrow 2 + 2\ln x_1 + 1.5\ln x_2 + 1.28\ln x_3$

$E(\ln w) = 2 + 2E(\ln x_1) + 1.5E(\ln x_2) + 1.28(\ln x_3)$

$E(\ln w) = 2 + 2\times 4 + 1.5\times 3 + 1.28\times 2$

$E(\ln w) = 17.06$

$Var(\ln w) = 0 + 4\sigma^2(\ln x_1) + 2.25\,\sigma^2(\ln x_2) + 1.28^2 \ln x_3$

$= 4\times 1 + 2.25\times 1 + 1.28^2\times 0.5$

$= 7.0692.$

$P(2 < w < 8) = 0.9$

$$\Rightarrow \quad \cancel{\mu - \ln} \quad \Phi\left(\frac{\mu - \ln r}{\sigma}\right)^{-1} = \Phi \; 0.90$$

$$\frac{\mu - \ln r}{\sigma} = 0.95$$

$$\Rightarrow \frac{\mu - \ln r}{\sigma} = 1.66$$

$$\Rightarrow \frac{17.06 - \ln r}{7.069} = 1.66$$

$$\ln r = 5.325 = 205.40$$

$$\text{Median} = e^{\mu} = e^{17.06}$$

Q8 $$e^{\frac{-1}{2(1-\rho^2)}\left(\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\left(\frac{x_1-\mu_1}{\sigma_1}\right)\left(\frac{x_2-\mu_2}{\sigma_2}\right) + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2\right)}$$

$$\frac{1}{2(1-\rho^2)} = \frac{2}{3}$$

$$\frac{3}{4} = 1-\rho^2 \qquad \rho = 1/2$$

$$\mu_1 = 0 \quad \sigma = 1$$

$$\rho = 1/2 \qquad P(-1 < x < 1) \mid Y = 1$$

$$\cancel{\mu_x} = \Rightarrow \quad \mu_x = 0 + \frac{1}{2}\left(\frac{\sigma_x}{\sigma_y}\right)(y-1)$$

$$\mu_1 \; \frac{1}{2}, \quad \sigma = \sigma^2(1-\rho^2)$$

$$\Rightarrow 1\left(1-\frac{1}{4}\right) = \frac{3}{4}.$$

$$-\frac{1\,n-\mu}{\sigma} \quad -\frac{1-\mu}{\sigma} \quad \frac{1-\mu}{\sigma}$$

$$1-\frac{1}{2}$$

$$\Rightarrow \left(\frac{-1-\frac{1}{2}}{\sqrt{3}/2}\right) + \phi\left(\frac{1-\frac{1}{2}}{\sqrt{3}/2}\right)$$

$$\Rightarrow \phi\left(\frac{1}{2}\times\frac{2}{\sqrt{3}}\right) - \phi\left(-\frac{1}{2}\times\frac{2}{\sqrt{3}}\right)$$

$$\cancel{\phi\left(2/3\right) - \phi}$$

$$\phi\left(\frac{1}{\sqrt{3}}\right) - \phi\left(\frac{-1}{\sqrt{3}}\right) = 2\phi(0.577) - 1$$

$$\Rightarrow 2\times 0.7190 - 1$$

$$-\frac{3}{2}\times\frac{2}{\sqrt{3}}\,(=-\sqrt{3}), \qquad -\frac{1}{2}\times\frac{2}{\sqrt{3}}$$

$4 \cdot \text{var}X + 9\,\text{var}y$

$$\Rightarrow 4\cdot\frac{3}{4} + 9\cdot\frac{3}{4} = \frac{27}{4} + 3 = \frac{27+12}{4} = \frac{39}{4}$$

Q9 $\mu_A = 20 \quad \sigma_A^2 = 0.03$

$\mu_B = 14 \quad \sigma_B^2 = 0.01$

$$33.6 < A+B < 34.4$$

$$\Rightarrow \frac{33.6-34}{0.02}$$

$\Rightarrow \mu(A+B) = 34$

$\sigma_B^2 = 0.04$

$$\frac{34.4-34}{0.02}$$

$$\phi(2) - \phi(-2)$$

$$2\phi(2) - 1 = 0.95 \qquad \frac{-0.4}{0.02} \quad 2.\ \frac{0.4}{2} \quad 2.1$$

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$E(x / y=y) \quad P(x \mid Y=y) = \frac{\int_0^{\infty} f(x,y)\, dx}{\int f(x,y)}$

$E(x) = \int x \cdot \frac{f(x,y)}{f(y)}\, dx$

$~~E(x)~~ \; E(x<2 / y=y)$

$\Rightarrow \int_0^2 x \cdot \frac{f(x,y)}{f_1(y)}\, dx$

Q3 $\quad f(x,y) = \frac{1}{8}(6-x-y) \quad 0<x<2$

$2<y<4$

$f_x(x,y) = \int_2^4 \frac{1}{8}(6-x-y)\, dy$

$\Rightarrow \frac{1}{8}\left[6y - xy - \frac{y^2}{2}\right]_2^4$

$\Rightarrow \frac{1}{8}\left[24 - 4x - 8 - (12 - 2x - 2)\right]$

$\Rightarrow \frac{1}{8}\left[24 - 4x - 8 - 12 + 2x + 2\right]$

$\Rightarrow \frac{1}{8}\left[6 - 2x\right] = \frac{3-x}{4}$

$f_y(x,y) = \int_0^2 \frac{1}{8}(6-x-y)\, dx$

$\Rightarrow \frac{1}{8}\left[6x - \frac{x^2}{2} - yx\right]_0^2 = \frac{5-y}{4}$

$\frac{1}{8}\left[12 - 2 - 2y\right] = \frac{10-2y}{8}$

$$E(Y/X=x) = \int_2^4 y \frac{f(x,y)}{f_x}\, dy$$

$$\Rightarrow \frac{\int_2^4 y \cdot \frac{1}{8}(6-x-y)\, dy}{\frac{3-x}{4}}$$

$$\Rightarrow \int_2^4 \frac{y}{2(3-x)}(6-x-y)\, dy \Rightarrow \frac{1}{2(3-x)} \int_2^4 y(6-x-y)\, dy$$

$$\Rightarrow \frac{1}{2(3-x)} \left[\frac{6y^2}{2} - \frac{xy^2}{2} - \frac{y^3}{3} \right]_2^4$$

$$\Rightarrow \frac{1}{2(3-x)} \left[3 \times 16 - 8x - \frac{64}{3} - \left(12 - 2x - \frac{8}{3}\right) \right]$$

$$\Rightarrow \frac{1}{2(3-x)} \left[48 - 8x - \frac{64}{3} - 12 + 2x + \frac{8}{3} \right]$$

$$\frac{1}{2(3-x)} \left[36 - 6x + \frac{-56}{3} \right]$$

$$\cancel{\frac{1}{2(3-x)} \left[60 - 6x \right] = \frac{6[10-x]}{2(3-x)}}$$

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$$\text{cov}(xy) = E(xy) - E(x) \cdot E(y)$$

$$\int_2^4 \int_0^2 xy \frac{1}{8}(6-x-y)\,dx\,dy$$

$$\frac{1}{8}\int_2^4 dy \int_0^2 6xy - x^2y - xy^2\,dx$$

$$\frac{1}{8}\int_2^4 dy \left[3x^2y - \frac{x^3y}{3} - \frac{x^2y^2}{2}\right]_0^2$$

$$\frac{1}{8}\int_2^4 dy \left[12y - \frac{8y}{3} - \frac{4y^2}{2}\right]$$

$$\frac{1}{8}\int_2^4 dy \left[\frac{28y}{3} - \frac{4y^2}{2}\right]$$

$$\frac{1}{8}\int_2^3 \left[\frac{28y}{3} - 2y^2\right] dy \Rightarrow \frac{1}{8}\left[\frac{28y^2}{6} - \frac{2y^3}{3}\right]$$

$$\frac{1}{8}\left[\frac{28}{6} \cdot 9 - \frac{54}{3} - \left(\frac{28 \cdot 4}{6} - \frac{2 \cdot 8}{3}\right)\right]$$

$$\frac{1}{8}\left[\frac{126}{3} - \frac{54}{3} - \frac{56}{3} + \frac{24}{3}\right]$$

Q7

P

Q7

Q4) $\mu_p = 28.4$, $\mu_{H} = 31.6$ $\sigma_1 = 46.24$
$\sigma_2 = 54.76$ $\rho_{12} = 0.82$

$X, Y \sim N(\mu_v, \sigma_v)$

$P(X > 30)$

$$\boxed{X = 28.4 + \frac{46.24}{54.76} \cdot (y -} \rightarrow \text{when } y = i$$

$$P\left(Z > \frac{X-\mu}{\sigma}\right) = 1 - P\left(Z < \frac{X-\mu}{\sigma}\right)$$

$$\Rightarrow 1 - P\left(Z < \underline{30 - 28.4}\right.$$

$P(Y > 35 / X = 30)$

$$Y = \mu_y + \left(\frac{\sigma_y}{\sigma_x}\right)\rho \underline{+ (x - \mu_x)}$$

35-

Q7

(left page fragments:) $-\frac{2y^3}{3}\Big]^3$ $-\frac{2.8}{5}\Big)$

Q7

$$f_{(x,y)}(x,y) = \frac{1}{y} e^{-\left(y+\frac{x}{y}\right)} \quad x>0 \quad y>0$$

$$f_y(x,y) = \int_0^\infty \frac{1}{y} e^{-\left(y+\frac{x}{y}\right)} dx$$

$$\Rightarrow \frac{1}{y}\Big[\int_\infty e^{-y} \cdot e^{-\frac{x}{y}} dx$$

$$\frac{1}{y} e^{-y} \cdot \int_0 e^{-x/y} dx$$

$$\Rightarrow \frac{1}{y} e^{-y} \times \cancel{y} e^{-x/y} \Big|_0^\infty$$

$$\frac{1}{\cancel{y}} \cancel{y} e^{-y} \cdot y \; e^{-x/y} \Big|_0^\infty \Rightarrow e^{-y}$$

$$\frac{f(x,y)}{f_y(y)} = \frac{\frac{1}{y} e^{-\left[y+\frac{x}{y}\right]}}{e^{-y}} = \frac{1}{y} e^{-x/y}$$

$$E(y) = \int_0^\infty y \, f_y(x,y) \, dy$$

$$\Rightarrow \int_0^\infty y e^{-y} dy$$

$$\Rightarrow -ye^{-y} - \int -e^{-y} dy$$

$$\Rightarrow -ye^{-y} - e^{-y} \Big|_0^\infty = +1 \text{ Ans.}$$

$E(x) = E[E(x/y)] = E(x)$

$E[\cancel{x/y}] \Rightarrow f(x/y) = \dfrac{f_x(x,y)}{f_y(y)}$

$\Rightarrow \dfrac{\frac{1}{y} e^{-(y+\frac{x}{y})}}{e^{-y}} = \frac{1}{y} e^{-x/y}$

$E(x/y)$

first we need to find conditional Probability of x given y. then the expectat.

$E(x/y) = \int_0^\infty \frac{x}{y} e^{-x/y} \, dx$

$\Rightarrow \frac{1}{y} \int_0^\infty x e^{-x/y} \, dx$

$\Rightarrow \frac{1}{y} \left[-yx e^{-x/y} \right] - \int -y \, y x e^{-x/y}$

$\Rightarrow \frac{1}{y} \left[-yx e^{-x/y} - y^2 x e^{-x/y} \right]_0^\infty$ $\quad E(x)=1$

$\Rightarrow -x e^{-x/y} - y e^{-x/y} \Big|_0^\infty$

$\Rightarrow 0 - 0 - (-y - 0)$

$\Rightarrow y$

$\cancel{E(x) = E(E(1))}$

$var(y) = E(y^2) - (E(y))^2$

$E(y^2) = \int_0^\infty y^2 e^{-y} \, dy$

$\Rightarrow -y^2 e^{-y} - \int [2y - e^{-y}]$

$= -y^2 e^{-y} - 2 \int y e^{-y} \, dy$

$\Rightarrow -y^2 e^{-y} - 2 \left[-y e^{-y} - \int -e^{-y} \right]$

$= -y^2 e^{-y} - 2 \left[-y e^{-y} - e^{-y} \right]_0^\infty$

$\Rightarrow -y^2 e^{-y} + 2y e^{-y} + 2e^{-y} \Big|_0^\infty$

$\Rightarrow 2$

$Var(y) = 2 - 1 = 1$

$E(x^2/y) = x^2$ | $x < \cancel{0.5}\ 3/4 \,/\, y = 1/6$ |

$E\left(y/x = 1/6\right)$

Q $f(x,y) = \frac{1}{4}(1+xy)$ $|x|<1$; $|y|<1$

$U = x^2$, $V = y^2$

$x = \pm\sqrt{U}$ $\frac{dx}{dU} = \frac{1}{2\sqrt{U}}$ $U = x^2$, $V = y^2$

$y = \pm\sqrt{V}$

$\frac{dx}{dV} = 0$

$\frac{dy}{dV} = \frac{1}{2\sqrt{V}}$ $\frac{1}{4\sqrt{UV}}$

$f(x,y) \Rightarrow \left(\frac{1+xy}{4}\right) \cdot \frac{1}{4\sqrt{UV}} \Rightarrow \frac{1}{2\sqrt{UV}}\left(1+\sqrt{UV}\right)$

$0 < U < 1$

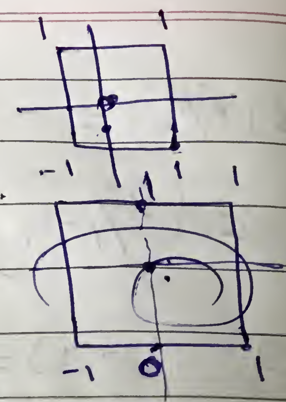
$0 < V < 1$

$\cancel{f(U,V) = \frac{1}{2\sqrt{UV}}\left(1+\sqrt{UV}\right)}$

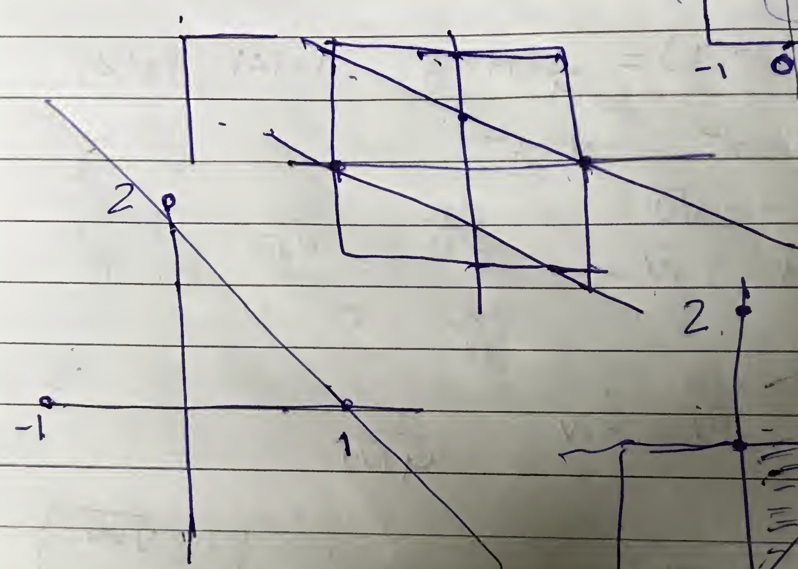
$\frac{1+\sqrt{UV}}{8\sqrt{UV}} \Rightarrow \frac{1}{8\sqrt{UV}} + \frac{1}{8}$

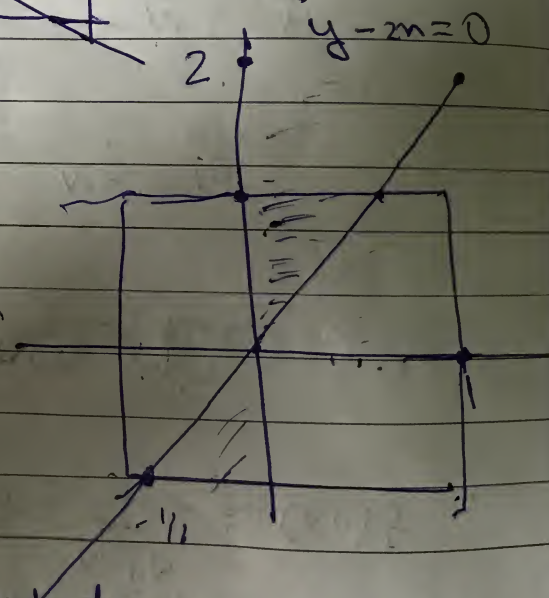
$f(x,y) = \frac{1}{4}(1+xy)$

$$f(x,y) = \frac{1}{4}(1+xy)$$



$P(2x<y)$





$$2 \times \int_0^1 \int_0^{1/2} \frac{1}{4}(1+xy)\,dx\,dy$$

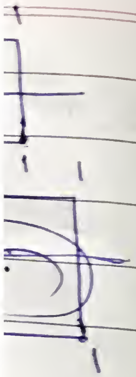
$$\frac{1}{8} + \frac{1}{64} = \frac{9}{64}$$

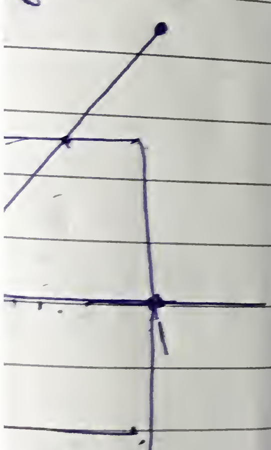
$$\frac{9}{32}$$

$$\Rightarrow \frac{1}{4}\, x + \frac{x^2 y}{8}\Big|_0^{1/2}$$

$$\frac{y}{8} + \frac{y^2}{64}\Big|_0^1$$

$$\Rightarrow \frac{1}{4}\left(\frac{1}{2} + \frac{y}{8}\right) \Rightarrow \frac{1}{8} + \frac{y}{32}\,dy$$





$= \frac{9}{64}$

2

dy

$$\int_{-1/2}^{1/2}\int_{-1}^{2x} \frac{1+xy}{4} + \int_{1/2}^{1} \frac{1+x}{4}$$

$$\Rightarrow \frac{1}{4}\int_{-1/2}^{1/2} \left(y + \frac{xy^2}{2}\right)_{-1}^{2x}$$

$$\frac{1}{4}\left[2x + 2x^3 - \left(-1 + \frac{x}{2}\right)\right]$$

$$\Rightarrow \frac{1}{4}\left[2x + 2x^3 + 1 - \frac{x}{2}\right] = \frac{1}{4}\left[\frac{3x}{2} + 2x^3 + 1\right]$$

$$\int_{-1/2}^{1/2} \frac{1}{4}\left[\frac{3x}{2} + 2x^3 + 1\right]$$

$$\frac{1}{4}\left[\cancel{\frac{3x^2}{4}} + \cancel{\frac{2x^4}{4}} + x\right]_{-1/2}^{1/2}$$

$$\frac{1}{4}\left[1\right] = 1/4$$

$$\Rightarrow \frac{1+x}{4}\,dx$$

$$\frac{1}{4}\left[x + \frac{x^2}{2}\right]_{1/2}^{1}$$

$$\Rightarrow \frac{1}{4}\left[1 + \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} - \frac{1}{8}\right]$$

$$\Rightarrow \frac{1}{4}\left[\frac{7}{8}\right] = 7/32$$

$$\frac{1}{4} + \frac{7}{32} = \frac{8+7}{32} \Rightarrow 15/32$$

$$\int_{1/2}^{1}\int_{-1}^{1} \frac{1+xy}{4}\,dy\,dx$$

$$\Rightarrow \frac{1}{4}\left[y + \frac{xy^2}{2}\right]_{-1}^{1}$$

$$\frac{1}{4}\left[1+\frac{x}{2} - \left[-1+\frac{x}{2}\right]\right]$$

$$\frac{1}{4}\left[1+\cancel{\frac{x}{2}}+1-\cancel{\frac{x}{2}}\right] \Rightarrow \frac{1}{2}$$

$$\frac{x}{2}\Big|_{1/2}^{1} \Rightarrow \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$$

d

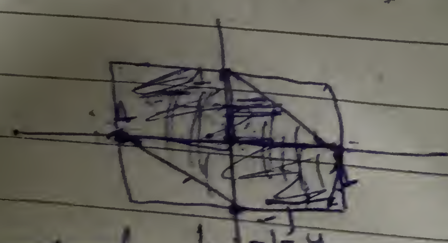
$P(|x+y|<1)$

$|x+y| = 1$

$-1 < x+y < 1$

$x+y = -1$

$x+y = 1$



$$\int_0^1\int_0^{1-y} \frac{1+xy}{4} \quad \left[\int_0^1\int^{-1-y} \frac{1+xy}{4}\right.$$

$$+\int_0^{-1}\int_0^{1-x} + \int_0^1\int_0^{-1-x}$$

$-(1+y)$

$$\Rightarrow \frac{1}{4}\left[x + \frac{x^2}{\cancel{y}2}\right]_0^{1-y}$$

$$\cancel{\frac{1}{4}\left[1-y+\frac{(1-y)^2}{y}\right]} = \frac{1}{4}\left[\cancel{1-y+\frac{y^2+1-2y}{y}}\right]$$

$$\frac{1}{4}\left[x+\frac{x^2 \cdot y}{2}\right]_0^{1-y}$$

$$\frac{1}{4}\left[1-y+\frac{(1-y)^2 y}{2}\right]$$

$$\frac{1}{4}\left[y-\frac{y^2}{2}+\frac{1}{2}\left[\frac{y^4}{4}-\frac{2y^3}{3}+\frac{y^2}{2}\right]\right]_0^1$$

$$\frac{1}{4}\left[1-\cancel{\frac{1}{2}}+\frac{1}{8}-\frac{1}{3}+\cancel{\frac{1}{2}}\right]$$

$$\frac{1}{4}\left[\frac{1}{3}+\frac{1}{8}=\frac{19}{24}\right]=\frac{19}{96}$$

$$\frac{1}{4}\left[x+\frac{x^2 y}{2}\right]_0^{-(1+y)}$$

$$\frac{1}{4}\left[-(1+y)+\frac{(1+y)^2 \cdot y}{2}\right]$$

$$\frac{1}{4}\left[-y-\frac{y^2}{2}+\frac{1}{2}\left[y^3+2y^2+y\right]\right]$$

$$\frac{1}{4}\left[-y-\frac{y^2}{2}+\frac{y^4}{8}+\frac{y^3}{3}+\frac{y^2}{4}\right]_0^1$$

$$\left[\frac{-36+19}{24}\right]$$

$$-(1+y)$$

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$\frac{19}{96}$ $\frac{-17}{96}$ $\int_0^1 \int_0^{-1} \left(\frac{1+xy}{4}\right) dx$

$\int_0^1 \frac{x+x^2y}{4} \Big|_0^{-1} \Rightarrow \frac{-1+y}{4}\, dx$

$-y+\frac{y^2}{2} \Big|_0^1 \Rightarrow \left(-1+\frac{1}{2}\right)\frac{1}{4}$

$\Rightarrow \frac{-1}{8}$

$\int_0^{-1} \int_0^1 \frac{x+x^2y}{4} \Big|$

$\Rightarrow \int_0^{-1} \left[\frac{1+y}{4}\right] \Rightarrow \frac{y+\frac{y^2}{2}}{4} \Big|_0^{-1}$

$\Rightarrow \frac{-1+\frac{1}{2}}{4} \Rightarrow 1/8$

$\cancel{\frac{1}{2}+\frac{17}{24}+\frac{96}{96} = \frac{48+68+}{96}}$ 1

$\frac{19}{96}-\frac{17}{24}+\frac{1}{4} = \frac{96-68+24}{96} \Rightarrow \frac{120-68}{96}$

$\Rightarrow \frac{52}{96}$ $\cancel{\frac{13}{24}}$